Incoherent white light solitons in logarithmically saturable noninstantaneous nonlinear media

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We analytically demonstrate the existence of white light solitons in logarithmically saturable noninstantaneous nonlinear media. This incoherent soliton has elliptic Gaussian intensity profile, and elliptic Gaussian spatial correlation statistics. The existence curve of the soliton connects the strength of the nonlinearity, the spatial correlation distance as a function of frequency, and the characteristic width of the soliton. For this soliton to exist, the spatial correlation distance must be smaller for larger temporal frequency constituents of the beam.

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I. INTRODUCTION

The propagation of incoherent light in noninstantaneous nonlinear media and the associated nonlinear effects, such as spatially incoherent solitons, have received considerable attention in recent years [1–21]. It all started with the experiment of Mitchell *et. al.* [1] which demonstrated the existence of optical spatial solitons made of partially spatially incoherent light. The spatially incoherent beam was generated by passing laser light through a rotating diffuser [1]. The result was intriguing and called for further research, since until then solitons were considered solely as coherent entities. Another experiment by Mitchell and Segev [2] went one step further and demonstrated solitons made of incoherent "white" light by using light emitted from an incandescent light bulb [2]. The experimental results from Refs. [1] and [2] were followed by a great deal of theoretical efforts aimed at understanding solitons made of incoherent light [3–21]. Importantly, in some cases closed-form analytical solutions for partially spatially incoherent quasimonochromatic solitons were found. This is the case for the logarithmically saturable nonlinearity, where closed-form solutions were first found by using coherent density theory [10], and subsequently by modal theory [11], and mutual coherence function theory [12,13]. Analytic solutions for spatially incoherent solitons were also found in Kerr-like media [14–19]. However, all of these theoretical studies considered only spatially incoherent, but temporally coherent (quasimonochromatic) light. Therefore, they are unable to describe both temporally and spatially incoherent solitons, which were observed experimentally in Ref. [2].

Temporally and spatially incoherent solitons were treated theoretically for the first time in a recent study [22]. The characteristic features of these solitons, such as properties of the temporal power spectrum and spatiotemporal coherence properties, were analyzed by using numerical methods [22].

In this paper, we present the closed-form solution representing temporally and spatially incoherent solitons. More specifically, in logarithmically saturable noninstantaneous nonlinear media, we find an analytic solution representing a family of such incoherent solitons. These incoherent solitons have elliptic Gaussian intensity profile, and elliptic Gaussian spatial correlation statistics. The existence curve of such a soliton connects the strength of the self-focusing, the spatial correlation distance at a particular frequency, and the characteristic width of the soliton. From the existence curve it follows that this soliton exists only when the spatial correlation distance is smaller for higher frequency constituents of the light.

II. THE MUTUAL SPECTRAL DENSITY THEORY

We begin with a brief review of the mutual spectral density approach utilized to describe the evolution of temporally and spatially incoherent light in noninstantaneous nonlinear media [23]. The physical system under consideration is as follows: The source of light emits spatially and temporally incoherent cw (not pulsed) light. The temporal power spectrum of the light is broad, and contained within some interval $[\omega_{min},\omega_{max}]$. For example, the light source used in Ref. [2] was an incandescent light bulb, and the width of the temporal power spectrum was $|\omega_{max}-\omega_{min}|/\omega_0 \sim 0.3$, where $\omega_0 =(\omega_{max}+\omega_{min})/2$. Although the temporal power spectrum is finite, due to the fact that it is broad, we refer to such light as white light [24]. Furthermore, in the spirit of Ref. [2] we call the solitons made of such light white light solitons. The beam formed from temporally and spatially incoherent light enters the noninstantaneous nonlinear medium. Due to the noninstantaneous response of the medium, the induced nonlinear index of refraction is unable to follow fast phase fluctuations of incoherent light, but responds only to the time-averaged intensity I. The time-averaged intensity I is in temporal steady state: $\partial I/\partial t=0$; the time average is taken over the response time of material. The dynamical equation(s) that are used describe the evolution of time-averaged quantities (i.e., statistically averaged quantities) along the propagation axis z (not in time t).

By assuming linear polarization of the light, the instantaneous electric field is described by a complex amplitude $\tilde{E}(x,y,z,t)$, and the spatiotemporal coherence properties of the light are described by the mutual coherence function [25],

$$\Gamma(\mathbf{R}_1,\mathbf{R}_2;\tau)=\langle\tilde{E}^*(\mathbf{R}_2,t_2)\tilde{E}(\mathbf{R}_1,t_1)\rangle =\frac{1}{2\pi}\int_0^\infty d\omega\Gamma_\omega(\mathbf{R}_1,\mathbf{R}_2)e^{-i\omega\tau}, \quad (1)$$

where $\tau=t_1-t_2$, and $\Gamma_\omega(\mathbf{R}_1,\mathbf{R}_2)$ denotes the mutual spectral density [25]. Brackets $\langle\cdots\rangle$ denote the time average over the response time of the material. In photorefractives, the response time can be as long as 0.1 s. The mutual coherence function describes the correlation statistics between the electric field values at two points $(\mathbf{R}_1,t_1)$ and $(\mathbf{R}_1,t_2)$ that are separated in space and time [25]. We are interested in the correlation statistics of the field between points upon the transverse cross section of the beam. Transverse cross section is perpendicular to the propagation z axis; let $\mathbf{r}_1$ and $\mathbf{r}_2$ denote the coordinates in this plane, i.e., $\mathbf{R}_{1,2}=\mathbf{r}_{1,2}+z\mathbf{k}$, where $\mathbf{k}$ denotes the unit vector of the z axis. The correlation statistics in this plane is described by the mutual spectral density $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)=\Gamma_\omega(\mathbf{r}_1+z\mathbf{k},\mathbf{r}_2+z\mathbf{k})$. Under the paraxial approximation, the evolution of B_ω is governed by an integrodifferential equation [23]

$$\frac{\partial B_\omega}{\partial z}-\frac{i}{2k_\omega}[\triangle_\perp^{(1)}-\triangle_\perp^{(2)}]B_\omega =\frac{ik_\omega}{n_0}\{\delta n(I(\mathbf{r}_1,z))-\delta n(I(\mathbf{r}_2,z))\}B_\omega(\mathbf{r}_1,\mathbf{r}_2,z), \quad (2)$$

where $I(\mathbf{r},z)=1/2\pi\int_0^\infty d\omega B_\omega(\mathbf{r},\mathbf{r},z)$ denotes the time-averaged intensity; the response of the material is $n^2(I)=n_0^2+2n_0\delta n(I)$, where n_0 and $\delta n(I)$ denote the linear and nonlinear contributions, respectively, to the refractive index; and $k_\omega=\omega n_0/c$.

In deriving Eq. (2) we have assumed that the medium is dispersionless, i.e., the linear part of the refractive index n_0 is independent of frequency. Since the term $\delta n(I)$ that couples all frequencies is independent of time $\partial\delta n(I)/\partial t=0$, and since Eq. (2) is in the frequency domain, dispersion can be included by substituting $n_0\to n_0(\omega)$. In this paper, we neglect the effect of dispersion to allow for analytical calculations. Since the light is cw and the induced index of refraction $\delta n(I)$ is independent of time, if $n_0(\omega)$ does not vary significantly over the frequency span $[\omega_{min},\omega_{max}]$, dispersion effects are negligible.

The mutual spectral density $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ contains information on both intensity and spatial coherence properties of light at frequency ω. The information on coherence properties only is extracted by normalizing $B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ [25],

$$\mu_\omega(\mathbf{r}_1,\mathbf{r}_2,z)=\frac{B_\omega(\mathbf{r}_1,\mathbf{r}_2,z)}{\sqrt{B_\omega(\mathbf{r}_1,\mathbf{r}_1,z)B_\omega(\mathbf{r}_2,\mathbf{r}_2,z)}}. \quad (3)$$

The quantity $\mu_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ is referred to as the complex coherence factor at frequency ω [25,26]. The spatial correlation distance at frequency ω is determined by the characteristic width of $\mu_\omega(\mathbf{r}_1,\mathbf{r}_2,z)$ [25].

For the analysis presented henceforth, it is convenient to introduce new coordinates

$$\mathbf{r}=\frac{\mathbf{r}_1+\mathbf{r}_2}{2} \quad\text{and}\quad \boldsymbol{\rho}=\mathbf{r}_1-\mathbf{r}_2.$$

Equation (2) in terms of new coordinates reads

$$\frac{\partial B_\omega}{\partial z}-\frac{i}{k_\omega}\left[\frac{\partial^2}{\partial r_x\partial\rho_x}+\frac{\partial^2}{\partial r_y\partial\rho_y}\right]B_\omega =\frac{ik_\omega}{n_0}\left\{\delta n\left[I\left(\mathbf{r}+\frac{\boldsymbol{\rho}}{2},z\right)\right]-\delta n\left[I\left(\mathbf{r}-\frac{\boldsymbol{\rho}}{2},z\right)\right]\right\}B_\omega(\mathbf{r},\boldsymbol{\rho},z); \quad (4)$$

the spatial vector $\mathbf{r}=(r_x\mathbf{i}+r_y\mathbf{j})/2$ is used to describe the variations of the time-averaged intensity in space, whereas the difference vector $\boldsymbol{\rho}=\rho_x\mathbf{i}+\rho_y\mathbf{j}$ is used to describe the correlation between phases at two different spatial points from the transverse cross section of the beam. We utilize Eq. (4) as the starting point to find white light solitons, such as those observed in Ref. [2].

III. SOLITONS IN THE LOGARITHMICALLY SATURABLE NONLINEARITY

We consider the following model for the nonlinear refractive index: $\delta n(I)=\kappa\ln(I/I_t)$ [10–13,27]. The coefficient $\kappa>0$ specifies the strength of the nonlinearity, while I_t is the threshold intensity. Although this model nonlinearity differs from the photorefractive screening nonlinearity [28] in which white light solitons were observed experimentally [2], it does provide a platform upon which we can find analytical solutions that yield insight into the realistic physical process. In fact, previous studies of coherent solitons [27] as well as of spatially incoherent quasimonochromatic solitons [10–13] have used this model nonlinearity to gain valuable insight. In this spirit, we use the logarithmic nonlinearity that yields closed-form solutions, highlighting important features of incoherent white light solitons.

To seek steady state solutions we require that both the intensity profile and the spatiotemporal coherence properties of the beam do not change during propagation, i.e., we require $\partial B_\omega/\partial z=0$. Since quasimonochromatic spatially incoherent solitons with elliptic Gaussian intensity profiles and correlation statistics were previously found in logarithmically saturable [11] and realistic saturable [21] nonlinear media, we seek for stationary wave solutions in the form

$$B_\omega(r_x,\rho_x,r_y,\rho_y)=A_\omega\exp-\left[\frac{r_x^2}{2R_x^2}+\frac{\rho_x^2}{2Q_x^2}+\frac{r_y^2}{2R_y^2}+\frac{\rho_y^2}{2Q_y^2}\right]. \quad (5)$$

Here A_ω denotes the spectral density of the light beam; the quantities R_x and R_y denote the characteristic width of the spatial soliton, whereas Q_x and Q_y are closely related to the spatial correlation distance of the incoherent light. When anzatz (5) is inserted into evolution equation (4) with $\partial B_\omega/\partial z=0$, and $\delta n(I)=\kappa\ln(I/I_t)$, it follows that quantities Q_x and Q_y must obey

$$Q_x=Q_y=\frac{1}{k_\omega}\sqrt{\frac{n_0}{\kappa}}=\frac{c}{\omega\sqrt{n_0\kappa}}=\frac{\omega_0}{\omega}Q_0, \tag{6}$$

where $Q_0=c\omega_0^{-1}/\sqrt{n_0\kappa}$ and ω_0 denotes the central frequency within the power spectrum. Quantities Q_x and Q_y are determined by the strength of the nonlinearity κ, frequency ω, the linear index of refraction n_0, and the speed of light c.

The spatial coherence properties for each frequency constituent of the beam can be found from the complex coherence factor [see Eq. (3)]:

$$\mu_\omega(r_x,\rho_x,r_y,\rho_y)=\Pi_{i=x,y}\exp\left\{-\left[\frac{1}{Q_0^2}\frac{\omega^2}{\omega_0^2}-\frac{1}{4R_i^2}\right]\frac{\rho_i^2}{2}\right\}=\exp\left[-\frac{\pi}{l_{s,x}^2(\omega)}\frac{\rho_x^2}{2}-\frac{\pi}{l_{s,y}^2(\omega)}\frac{\rho_y^2}{2}\right], \tag{7}$$

where $l_{s,x}(\omega)$ and $l_{s,y}(\omega)$ denote the spatial correlation distances at frequency ω in the x and y directions, respectively. These spatial correlation distances are determined by the characteristic widths of the complex coherence factor μ_ω [25]. From Eq. (7) it follows that the characteristic widths of the elliptic beam are connected to the spatial correlation distances $l_{s,x}(\omega)$ and $l_{s,y}(\omega)$ and the strength of the nonlinearity through

$$\frac{1}{l_{s,i}(\omega)}=\sqrt{\frac{n_0\kappa}{\pi c^2}\omega^2-\frac{1}{4\pi R_i^2}},\quad i=x,y. \tag{8}$$

Equation (8) is the existence curve for the white light solitons in the logarithmically saturable nonlinear medium. In the limit of temporally coherent (quasimonochromatic), but spatially incoherent solitons, we recover the solution for such solitons in logarithmic media [10–12]. In the limit of spatially and temporally coherent beams, we recover the solution for coherent solitons in logarithmically saturable nonlinear media [27].

From the existence curve we read that for white light solitons to exist, the spatial correlation distance must decrease for higher frequency constituents of the light. Figure 1 illustrates the functional dependence $l_{s,i}(\omega)$ as calculated from Eq. (8) for realistic parameter values. This result can be interpreted as follows. Optical spatial solitons occur when diffraction is exactly balanced by refraction (nonlinearity). White light solitons are made up of a continuum of frequencies (wavelengths). Through the nonlinear coupling $\delta n(I)$, every frequency constituent "see" the same self-induced waveguide, that is, the refraction "force" felt by every frequency constituent is the same. Consequently, to balance this refraction force, all frequencies have the same diffraction angle θ. If the size of the beam is several times larger than the spatial correlation distance, the diffraction angle θ is mainly governed by the degree of coherence. In that limit, θ is proportional to the ratio of the wavelength and the spatial correlation distance, $\theta\propto\lambda/l_{s,i}(\lambda)$ [20]. From this, we immediately obtain $l_{s,i}(\lambda)\propto\lambda$, which is exactly the result from Eq. (8) in the limit $R_i\gg l_{s,i}(\omega)$. Equation (8) is more accurate since it takes into account diffraction from (i) the finite size of the beam envelope and (ii) the spatial incoherence.

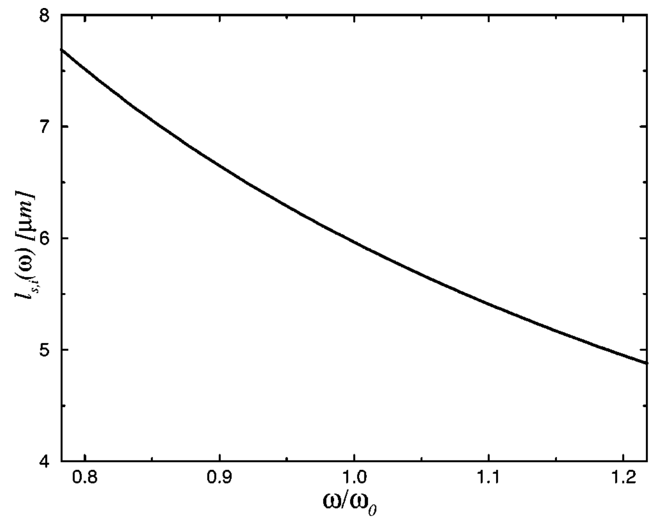

FIG. 1. The spatial correlation distance decreases with the increase of frequency. The value of $l_{s,i}(\omega)$ is calculated from Eq. (8) with the following parameters: $\kappa=0.0003$, $n_0=2.3$, $R_i=10$ μm, $\omega_0=3.44\times10^{15}$ Hz, which corresponds to the wavelength of 547 nm in vacuum.

From Eq. (8) it also follows that the characteristic widths should be larger than some threshold value,

$$R_i>\frac{1}{2}\frac{c}{\omega\sqrt{\kappa n_0}}. \tag{9}$$

Inequality (9) must be satisfied for every frequency ω within the spectrum. Suppose that the frequencies lie within the interval $[\omega_{min},\omega_{max}]$, where $\omega_{min}=\omega_0(1-\Delta)$ and $\omega_{max}=\omega_0(1+\Delta)$; Δ denotes the width of the temporal power spectrum, i.e., the degree of temporal coherence. Clearly, if

$$R_i>\frac{c}{2\omega_0(1-\Delta)\sqrt{\kappa n_0}}, \tag{10}$$

then inequality (9) is satisfied for all frequencies. This means that for the white light soliton to exist, its size must exceed a value imposed by the degree of temporal (in) coherence (Δ), and the strength of the nonlinearity.

From Eqs. (5) and (7) we see that the intensity profile and the spatial correlation statistics of this white light soliton are elliptic Gaussian functions. Figure 2 shows contour of the total intensity profile (thick long dashed line), and contours of the complex coherence factor at three representative frequencies $\omega_{min}=2.69\times10^{15}$ Hz (dotted line), $\omega_0=3.44\times10^{15}$ Hz (dashed-dotted line), and $\omega_{max}=4.19\times10^{15}$ Hz (solid line). From the μ_ω contours we see that higher fre-

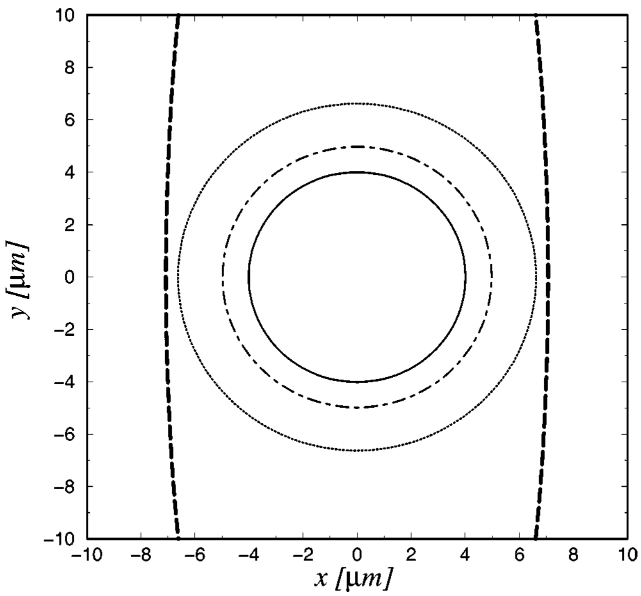

FIG. 2. The intensity structure and the complex coherence function of an incoherent white light soliton in logarithmic nonlinearity. The outer (thick) dashed curve is a contour plot of the intensity profile of the soliton $I(x,y)$ defined by $I(x,y)=I_0e^{-1}$ (I_0 is the intensity at the peak). The three internal ellipses are contours of the complex coherence factor μ_ω at frequencies $\omega_{min}=2.69\times10^{15}$ Hz (dotted line), $\omega_0=3.44\times10^{15}$ Hz (dashed-dotted line), and $\omega_{max}=4.19\times10^{15}$ Hz (solid line). These contours are defined by $\mu_\omega(x_1,y_1,x_2,y_2)=\mu_\omega(0,0,x,y)=e^{-1}$. Other parameters are $\kappa=0.0003$, $n_0=2.3$, $R_x=5$ μm, and $R_y=20$ μm. The contours of μ_ω have smaller area for higher frequencies, i.e., coherence area is smaller for higher frequency constituents of the soliton beam. The cross section of the complex coherence factor is an ellipse slightly stretched along the *x* axis, whereas the intensity profile ellipse is stretched along the *y* axis.

quency constituents of the beam have smaller coherence area, which is in accordance with Fig. 1. From Eq. (8) it also follows that $l_{s,i}(\omega)$ decreases with the increase of R_i. This means that major axis of the intensity profile ellipse is perpendicular to the major axes of the ellipses representing the complex coherence factors. Ellipses representing μ_ω from Fig. 2 are very slightly elongated along the *x* axis (they are almost circular) since parameter values there correspond to the case when $l_{s,i}(\omega)$ is one order of magnitude smaller than $2\sqrt{\pi}R_i$. In such a case, $l_{s,x}(\omega)\simeq\sqrt{n_0\kappa\pi^{-1}}\omega c^{-1}\simeq l_{s,y}(\omega)$ [see Eq. (8)]. The ellipses become more elongated when $l_{s,i}(\omega)$ is several times larger than $2\sqrt{\pi}R_i$, i.e., when the correlation distance is several times larger than the characteristic width of the soliton. When the value of one of the intensity profile semiaxes, say R_y, becomes infinitely large, i.e., when $R_y\to\infty$, the soliton from Eq. (5) becomes $(1+1)D$.

Note that the complex coherence function $\mu_\omega(r_x,\rho_x,r_y,\rho_y)$ depends only on the difference coordinates ρ_i. This means that the statistical fluctuations of light that forms the soliton obey a stationary random process [25]. However, in the context of spatially incoherent quasimonochromatic solitons, it has been shown that the complex coherence factor must, in general, depend on the spatial coordinates r_i. More precisely, the spatial correlation distance is also a function of r_i, and must, in general, increase at the tails of the soliton [20]. The soliton presented above does not contain this generic feature. This is attributed to the fact that the logarithmic nonlinearity is in fact approximation of the more realistic case, $\ln(1+I/I_t)$, in the limit when $I\gg I_t$ [10–13,27]; this limit is not satisfied at the soliton "tails." Hence, the model logarithmic nonlinearity has its limits in describing the realistic physical process: it gives useful information on the features of incoherent solitons except for the tails. For the features of white light solitons specifically, numerical simulations in realistic saturable nonlinearity shows that results highlighting the decrease of spatial correlation distance with the increase of frequency are generic for white light solitons [22].

Let us address the issue of soliton stability. This soliton satisfies the stability criterion $dI_0/dP>0$, where I_0 is the intensity at the soliton peak, while P denotes the power $P=\int\int dr_x dr_y I$ [29]. Since this criterion is established under quite general circumstances [29], we can conclude that white light soliton from Eq. (5) is stable. To support this view, let us analyze the propagation of the beam with characteristics slightly different from the white light soliton (5). We utilize the procedure from Ref. [12] for spatially incoherent, but temporally coherent solitons in logarithmic medium. We assume that the mutual spectral density is of the form

$$B_\omega(r_x,\rho_x,r_y,\rho_y,z)=a_\omega(z)\prod_{j=x,y}\exp\left[-\frac{r_j^2}{2s_j^2(z)}-\frac{\rho_j^2}{2q_j^2(z)}\frac{\omega^2}{\omega_0^2}+ir_j\rho_j\phi_\omega(z)\right], \tag{11}$$

where $\phi_\omega(z)$ and $a_\omega(z)$ denote the phase and the amplitude of the mutual spectral density, respectively; $s_j(z)$ denotes its width, and $q_j(z)$ is associated with the spatial correlation distance. When expression (11) is inserted in the evolution equation (4), we obtain a dynamical system for the set of coordinates $(a_\omega,s_j,q_j,\phi_\omega)$ [12]:

$$\frac{da_\omega(z)}{dz}=-\frac{1}{k_\omega}2\phi_\omega(z)a_\omega(z), \tag{12}$$

$$\frac{ds_j(z)}{dz}=\frac{1}{k_\omega}\phi_\omega(z)s_j(z), \tag{13}$$

$$\frac{dq_j(z)}{dz}=\frac{1}{k_\omega}\phi_\omega(z)q_j(z), \tag{14}$$

$$\frac{1}{k_\omega}\frac{d\phi_\omega(z)}{dz}=\frac{1}{k_{\omega_0}^2}\frac{1}{q_j^2(z)s_j^2(z)}-\frac{\phi_\omega^2(z)}{k_\omega^2}-\frac{\kappa}{n_0}\frac{1}{s_j^2(z)}. \tag{15}$$

It is straightforward to see that a fixed point of the dynamical system (12)–(15) is given by $q_j=Q_0=k_{\omega_0}^{-1}\sqrt{n_0/\kappa}$ and $\phi_\omega=0$, which are the conditions for the existence of white light soliton [see Eq. (6)]. To gain more insight into the stability of the soliton, let us observe the evolution of the dynamical system (12)-(15) from an initial condition slightly displaced from the soliton condition, $q_j(0)=Q_0+\delta q_j(0)$ and $\phi_\omega(0)=\delta\phi_\omega(0)$; without losing any generality we assume $s_j(0)=R_j$ and $a_\omega(0)=A_\omega$. From Eqs. (12)–(15) we see that the phase of the mutual spectral density ϕ_ω is proportional to the frequency, i.e.,

$$\phi_\omega(z)=k_\omega\phi_0(z)=\frac{\omega n_0}{c}\phi_0(z), \tag{16}$$

where $\phi_0(z)$ denotes the phase at the central frequency ω_0. From Eq. (12) it follows that $a_\omega(z)=a_\omega(0)a'(z)$, where $a'(z)$ is frequency independent. The quantities $a_\omega(z)$, $q_j(z)$, and $\phi_0(z)$ can be expressed in terms of the characteristic width $s_j(z)$. From Eqs. (13) and (14) it follows that $q_j(z)=q_j(0)/s_j(0)s_j(z)$. From Eqs. (12) and (13) we obtain $a_\omega(z)=a_\omega(0)[s_j(0)/s_j(z)]^2$. From Eq. (13) we see that $\phi_0(z)=s_j^{-1}(z)ds_j(z)/dz$, which together with Eq. (15) gives an ordinary differential equation for $s_j(z)$:

$$\begin{aligned}\frac{d^2s_j(z)}{dz^2}&=\frac{1}{k_{\omega_0}^2}\frac{s_j^2(0)}{q_j^2(0)s_j^3(z)}-\frac{\kappa}{n_0}\frac{1}{s_j(z)}\\&=F_j(s_j),\quad j=x,y.\end{aligned} \tag{17}$$

This is Newton's equation for a unit mass particle that feels a force $F_j(s_j)$. Finally, we can observe the evolution of a beam that is initially slightly displaced from the equilibrium: $q_j(0)=Q_0+\delta q_j(0)$, $|\delta q_j(0)|\ll Q_0$. The radius of the beam is $s_i(z)=R_i+\delta s_i(z)$, and the evolution of $\delta s_i(z)$ in the linearized regime $|\delta s_i(z)|\ll s_i(z)$ is given by

$$\begin{aligned}\frac{d^2\delta s_j(z)}{dz^2}&\simeq-\frac{2k_{\omega_0}\kappa^{3/2}\delta q_j(0)}{n_0^{3/2}R_j}-\frac{2\kappa}{n_0R_j^2}\delta s_j(z)\\&=-\frac{2k_{\omega_0}\kappa^{3/2}\delta q_j(0)}{n_0^{3/2}R_j}-\Omega_j^2\delta s_j(z),\quad j=x,y.\end{aligned} \tag{18}$$

The initial conditions for Eq. (18) are set by $s(z=0)=R_j$, and by the initial displacement of the phase $s'(z=0)=s_j(0)\delta\phi_0(0)=R_j\delta\phi_0(0)$, from which we obtain

$$\begin{aligned}s_j(z)=R_j\left(1-\frac{\delta q_j(0)}{Q_0}\right)+\frac{\delta q_j(0)}{Q_0}R_j\cos\Omega_jz\\+\frac{\delta\phi_0(0)}{\Omega_j}R_j\sin\Omega_jz.\end{aligned} \tag{19}$$

Thus, since the medium is self-focusing $\kappa>0$, the width of the beam $s_j(z)$ oscillates around the value $R_j(1-\delta q_j(0)/Q_0)$. If $\delta q_j(0)>0$, the spatial correlation distance is initially *larger* than the correlation distance of the soliton; consequently, diffraction tendency is initially smaller than self-focusing, and the radius oscillates around slightly smaller value than R_j [12]. If $\delta q_j(0)<0$, the spatial correlation distance is initially *smaller* than the correlation distance of the soliton; consequently, diffraction initially overcomes the nonlinearity, and the radius oscillates around slightly larger value than R_j [12]. Consequently, if the mutual spectral density is slightly different from the mutual spectral density of the soliton [Eqs. (5), (6), and (8)], it will oscillate close to it. The oscillations will be faster if the nonlinearity κ is larger, or if the characteristic radius R_j is smaller. To get an idea of the characteristic scales of the oscillations, for the parameters $\kappa=0.001$, $n_0=2.3$, and $R_j=20\ \mu$m, the frequency of oscillations is $\Omega_j=\sqrt{2\kappa n_0^{-1}R_j^{-2}}=1.47\ \text{mm}^{-1}$, and the period is $D_j=2\pi/\Omega_j=4.26$ mm. The oscillations will be periodic or quasiperiodic, since there are two characteristic frequencies of oscillation (one per each spatial coordinate).

IV. CONCLUSION

In summary, we have presented closed-form soliton solution representing spatially and temporally incoherent solitons in logarithmically saturable noninstantaneous nonlinear media. This incoherent soliton has elliptic Gaussian intensity profile, and elliptic Gaussian spatial correlation statistics. The existence curve of this soliton, Eq. (8), connects the strength of the nonlinearity κ, the spatial correlation distances as a function of frequency, $l_{s,x}(\omega)$ and $l_{s,y}(\omega)$, and the characteristic widths of the soliton R_x and R_y, respectively. From the existence curve it follows that for this soliton to exist, the spatial correlation distance must be smaller for larger frequency constituents of the light. Furthermore, its size must exceed a threshold imposed by the strength of the nonlinearity and the degree of temporal incoherence. The stability of the soliton follows from the criterion $dI_0/dP>0$, and from analyzing the oscillations of the mutual spectral density that is close to the mutual spectral density of the soliton. For future work on white light solitons, we envision ''dark'' and ''antidark'' white light solitons. The analysis of the spectral density at the darkest spot of a dark soliton and the vicinity of such spot seems to be an interesting problem in view of the recent study of universal pattern of colors near an isolated phase singularity [30]. Finally, the properties of interactions of white light solitons and other interesting problems have not been explored yet.

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